D21 – Janeček method

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Is it possible to find a voting system which maintains an equality of voting rights and at the same time gives greater impact to well-informed and engaged voters? Yes, it is possible. Our proposed voting system solves this apparent paradox. In addition to this principal benefit, it also enables voters to re-balance the political spectrum away from extremists and reduce the threat of dishonest or corrupt political actors.

1 Representative democracy

In 1947, Winston Churchill famously declared: "It has been said that democracy is the worst form of government, except all the others that have been tried." Is democracy really as weak, ineffective and unsatisfactory a form of government as it often appears? Might a better system than democracy exist? If not, does it mean that we, as a human society, must endure the ill effects of a system performing far under its potential?

It is the author's core conviction that only systems with a democratic anchor will work properly in the long run. As soon as the essential principle of equal suffrage is breached, the risk that the social system will eventually degenerate into some form of authoritarianism or dictatorship, or another regime which discriminates against and mistreats certain individuals or groups of citizens, is significant. Such a system will necessarily be unstable from a long-term perspective and will lead to major social conflicts.

Selection of high-quality political representation is a key factor for reaching the fairest and most effective functioning of society as a whole. In the case of most voting systems common today, the principle of "one man, one vote" as currently applied aided the rise of populism and extremism due to their embedded paradoxes. The result has been a series of problems within democratic societies, leading to recognizable signs of political disengagement, including public disillusionment, mistrust of the traditional political elite, political polarization, low turnout and the rise of political extremism, see [HT02]; [BNT17]; [OS96]; [NS03]. It is easier for paradoxes to emerge under this system since it prohibits voters from expressing any nuanced preferences other than for a single option, presumed to reflect their "choice" [Nur98]. The end result is that relatively easy

manipulation and control of the masses by demagogues, growing rates of voter abstention, and the risks of "tyranny of the majority" now pose an insoluble problem to 21st-century democracies. These risks are significant: how can we rise to the challenge?

This paper puts forth an alternative voting method as a possible solution to problems which can be attributed to the limitations of the current voting rules: rising political polarization, low voter turnout, and political disengagement. The lattermost issue is an especially crucial one. It is important to incentivize civic engagement by allowing citizens who invest their time and effort toward the political and economic development of society (we call them "engaged voters") to see their efforts repaid by a proportionally greater impact on the selection of political representatives. Hence, the basic idea of the voting system presented herein is that the voter can identify in a single ballot more than one preference by applying systematically capped multiple votes. The overall impact of engaged and informed voters, compared to disengaged or manipulated ones, will thus be higher, without compromising the standard of universal voting rights and equal access of parties and candidates to the political process.

It is also important to note that the power to cast more votes will give the electorate as a whole inherently greater motivation to consider their choices and deliberate with one another. In short, the system proposed here allows voters to express their preferences in a way countering the limitations of one-vote ballots under majority or plurality rules, which do not allow citizens to make political choices reflecting their multidimensional issue space, the presence of which has been acknowledged by scholars [Nur98]. By its inherent mathematical properties, this voting method will reinforce civic engagement, electoral participation, and social solidarity.

2 A new voting method: Description and features

D21 – Janeček method (further as D21) is neither a fully majoritarian nor proportional system nor a compromise between the two.[1] It permits voters to cast multiple votes that are either positive or negative. It can be applied in elections with multiple seats to be filled. The strength of D21 is mostly profound for two seats.

We first give a general definition of D21 in Section 2.1. For a comprehensive understanding of effects of D21, we describe it in the following steps:

- The effect of more votes (Section 2.2).
- The effect of more seats (Section 2.3).
- The effect of the minus-vote (Section 2.4).
- Positive campaigning (Section 2.5).

[1] Technically speaking, D21 is a semi-proportional system.

- Social choice criteria (Section 2.6).

2.1 General definition

Suppose we want to choose W winners out of $T \geq 4$ candidates. A voting system is D21 – Janeček method if and only if:

- Each voter is allowed to cast up to $P > W$ plus votes and up to $M \geq 0$ minus votes, where $P \geq 2M$ (i.e., number of plus votes has to be at least twice as large as the number of minus votes), and $P + M \leq T$. In most cases, we recommend $P + M \leq T/2$.
- Each voter can cast no more than one vote for any candidate.
- The number of plus-votes cast by each voter must be at least twice the number of minus-votes cast.
- Each vote has the same absolute weight (+1 or -1). The W candidates receiving the greatest sum of all votes win.

2.2 Effect of more votes

In this section we assume one seat per district ($W = 1$) with all voters having two plus-votes ($P = 2$) and zero minus votes ($M = 0$) for simplicity of exposition. The situation is analogous for multiple seats.

2.2.1 Sample situation

The effects in practical applications are analogous to the following model situation. We assume four "medium" candidates, i.e., democratic spectrum candidates, regardless of left- or right-wing positions on specific issues, who stand behind the basic guarantees of democratic pluralism, accountability and inclusion. We further have two polarizing candidates: An "extremist" is candidates who stands outside this democratic consensus and seeks to use political power to fundamentally alter democratic processes and institutions. A "populist" is candidates who, while nominally part of the democratic spectrum, bases his appeal on demagoguery and thus has no natural political allies outside his own party. For the purposes of the illustration, suppose the following:

- One right-wing extremist with 18% of preferences.
- Two right-wing "democratic spectrum" candidates, each with 16% of preferences.
- Two left-wing "democratic spectrum" candidates, each with 16% of preferences.
- One left-wing populist with 18% of preferences.

In the most common majoritarian system, first-past-the-post (FPTP)[2], either the left-wing populist or right-wing extremist would win; under a two-round system, both of them would qualify for the second round of voting. A key characteristic of D21 is an existence of a second vote which each voter may give to another candidate, as follows:

- A supporter of the right-wing extremist is likely to give his second vote to someone from the democratic right or withhold his second vote. Less probably, he will give his second vote to a candidate from the democratic left, and least probably to the populist candidate of the left-wing.
- A supporter of a democratic right-wing candidate will give his second vote most probably to the second democratic candidate of the right wing, less probably to a democratic left-wing candidate, least probably to the right-wing extremist.
- Supporters of left-wing candidates will behave accordingly, with designations of "right" and "left" reversed.

The results of voting can be expected as follows:

- The right-wing extremist will get just over 18 % of votes.
- The right-wing democratic candidate will get just over 32 % - he will get votes from his supporters (16 %) and most votes from supporters of the second democratic right-wing candidate (16 %), and some second votes from supporters of the right-wing extremist.
- Similarly, the left-wing democratic candidate will get just over 32 %.
- The left-wing populist will get just over 18 %.

The analysis shows that in our model one of the candidates of the "democratic spectrum" will be elected. In standard voting systems non-extremist candidates must compete with one another in a zero-sum contest for votes, meaning that right-wing or left-wing extremists or populists are more likely to win or advance to the second round to the detriment of candidates with broader appeal. A key characteristic of D21 is the fact that candidates with broader appeal are less likely to attack one another to their mutual detriment.

2.3 Effect of more seats

An additional weakness of majoritarian, single-winner districts is their suboptimal representativity – small parties can win a very significant number of votes without carrying a single legislative seat. The solution lies in the election of more than one candidate per voting district.

[2]In a first-past-the-post or winner-takes-all election, the winning candidate is the one who has received more votes than any others.

2.3.1 Version of D21 for parliament

For full use of the semi-proportional effects of D21 we specify a proposed voting system for parliament:

- Two seats per district ($W = 2$).
- A competing party will nominate one or two candidates.[3]
- Independent candidates may compete subject to other conditions (for example, upon gathering a certain number of signatures from eligible voters).
- Voters may cast up to four plus-votes ($P = 4$) and up to one minus-votes ($M = 1$)[4].

The additional seat, combined with the *effect of more votes*, has a significant and positive impact on representativity. Note that a smaller party has the option of nominating just one candidate, with the goal of attracting additional (third or fourth) votes from the supporters of larger or more well-established parties. A strong candidate nominated by a small party, or even an independent candidate, will thus have a far better chance of winning than under FPTP rules. As a result, new parties and ideas will have far greater opportunities to compete and succeed in the democratic process. Additionally, because of the effects of more seats and more votes, negative campaigning by any one candidate against any other is unlikely, see also Section 2.5.

We expect that in a parliament or legislative body elected under the D21 system, there will be more smaller parties represented than under a typical proportional system with minimum hurdles. At the same time, a strong party with broad electoral appeal might become even stronger, thanks to the "majoritarian" effect of our system, than it would under a proportional system. What kind of party will weaken? Parties with more narrow electoral appeal that are unable to obtain additional (3rd and 4th) votes, are likely to win fewer seats under D21 than under a typical proportional system.

With D21 we thus expect both higher representativity and innovation than under proportional voting systems, and, because of a more collaborative environment, an easier process of government formation. The electoral system will at once be more competitive and dynamic, and the formation of governments easier and more stable, than is the case under proportional voting systems.

2.4 Effect of the minus-vote

The third feature of D21 is the "minus-vote", which allows voters an even wider scope to express their preferences by letting them designate a candidate they

[3]This constraint is not essential: A party would be naturally disincentivized from nominating more than two candidates to create competition for itself.

[4]Under some specific circumstances, it may be desired to have no minus-vote at all (for example, where the electorate includes disfavored religious minorities).

do not want to see elected. A minus-vote and a plus-vote have the same absolute weight (-1 or +1). In itself, the availability of a minus-vote will probably result in a significantly higher rate of voter participation, especially in a climate of public skepticism or disapproval of current political leaders. Moreover, the existence of minus-votes further diminishes the electoral strength of polarizing candidates. Minus-votes provide an important means to filter notoriously corrupt and criminal actors from a political system where they are too often protected, for example by "hiding" in a party list and benefiting from the goodwill of voters toward their party and its other candidates.

2.4.1 Literature overview

It is important to note that Boehm first proposed the idea of Negative Voting (NV) in an unpublished essay in 1976. According to Boehm's approach, voters can either vote for one candidate or against one candidate, but cannot do both [BF05]. A candidate's "negative" votes would be subtracted from his or her "positive" votes to determine their net vote, and the candidate with the highest net vote would win. Examining Boehm's proposal, Brams, who developed an Approval Voting (AV)[5] method based on it, agreed that in three-candidate elections "negative votes may be uniquely advantageous." [Bra77] [Moh11] However, the idea of the "negative vote" was not developed further. Rather Brams developed AV based after reading Boehm's unpublished essay on the NV concept. [BF05] The soundness of "negative votes" was reexamined in the concept of "negative preferences" as an important subjective motivator for voting. Leef argues that the introduction of negative votes in United States presidential elections would increase voter participation, make elections more competitive with minor party representation and could ultimately lead to increased political engagement. [Lee]

A related mechanism is "best-worst" voting. This considers a choice design in which a person is asked to select not only the best but also the worst alternative. [GMM10] One of the advantages of the best-worst method is that it provides more information on preferences. [ML05] The results of Cahan's and Slinko's study of equilibria in the Hotelling-Downs spatial election model for the best-worst voting rules show that best-worst rules encourage a degree of policy moderation. They note that it discourages extremist platforms as they invite negative votes. [CS18] Similarly, Kang argues that "Prioritization of negative preferences in voting is more likely to avoid a choice that is intensely opposed by a segment of voters, even if it may produce a choice that is not most preferred by anyone." [Kan10]

Felsenthal's analysis of Combined Approval Voting (CAV) under which voters were given the option of voting against candidates in addition to voting for them or abstaining, shows that voters preferred CAV to regular approval

[5] AV was proposed in the 1970s by Weber, then a graduate student, disgruntled by the results of mayoral elections in Ithaca, New York. He described AV in the following way: "Each voter is allowed to cast a single vote for each of as many candidates as he or she wishes – that is, the voter votes for all candidates of whom the voter 'approves.'" [Web95]

voting (RAV). [Fel89] Hillinger claims: "A psychological shortcoming of AV is that it does not allow voters the satisfaction of explicitly voting against a disliked candidate." He argues that availability of "voting against" will give voters emotional satisfaction. [Hil05] Drawing on the earlier studies, Alcantud and Laruelle propose "Dis&approval Rule" with three voting options: approval (1), indifference (0), and disapproval (-1), claiming that it "enriches the options offered by approval voting by allowing voters to explicitly express disagreement with some (or all) candidates." [JL14] Another trichotomous procedure, which includes a disapproval option, is the Approval–Condorcet–Elimination (ACE). [Yil99] In addition, Baharad's and Nitzan's Single-Approval Multiple-Rejection (SAMR) scoring rule includes an option of rejection along with approval of the alternatives. [BN16]

There have been several cases where elements of dissatisfaction have been incorporated into actual election procedures. Some countries and US states have introduced a "None of the above" (NOTA) option to their ballots. [Ind] Damore et al. argue that though NOTA voting shares some features of abstention, it can be linked with dissatisfaction since casting a NOTA vote requires voters to register, turn out at the election poll and complete the ballot. [DWB12] Examples include that of Russia from 1993 to 2005, when citizens could vote "against all" candidates and parties at all elections including local and presidential ones. [MW08]

2.4.2 Anti-corruption effect of the minus vote

We assume a simple model with one voting district with two parties nominating their candidates. We denote the "right-wing" candidate as R and the "left-wing" candidate as L. Each party will nominate two candidates, one corrupt (-) and one honest (+). We will mark them as L^+, L^-, R^+, R^-. In an ideal case, the honesty of any individual candidate would be a decisive criterion for voters, and minus-votes would be given to corrupt candidates, with winners L^+ and R^+. In our model, we assume the worst possible scenario, where all voters of each party prefer the corrupt candidate of their own party to the honest candidate of the opposing party and, at the same time, they are capable of so-called tactical voting. Tactical voting means that voters will distribute their minus-votes in a manner intended to benefit their preferred candidate rather than register disapproval of another; thus, tactical voters may be willing to cast minus-votes against the honest candidate of the opposing party rather than the corrupt candidate of the opposing party, if they judge this to benefit their own preferred candidate.

Even in this worst-case scenario, the presence of minus-votes acts to purify the system from corrupt actors. The honest candidates L^+ and R^+ are elected as soon as the difference between the size of the right-wing and left-wing electorate is not too great. Specifically, L^+ and R^+ win as soon as the ratio between the number of right-wing voters n_r and the left-wing voters n_l is more than 3 to 4 and less than 4 to 3. In the case that voters do not vote tactically, i.e. where they give their minus-votes to the corrupt candidate of the opposing party, the

purifying effect happens for a ratio of votes between 1 to 2 and 2 to 1. See the Attachment 1 for a proof.

The conclusion above holds that both of the honest candidates win as soon as $n_r/(n_l + n_r) \in \left(\frac{3}{7}, \frac{4}{7}\right)$. Without the existence of a minus-vote, both candidates of one of the two parties will win. Therefore, the minus-vote brings significant purification of political representation. A real situation is of course more complicated than the model. Note, however, that the model assumption are very "pessimistic" in terms of preferences and tactics of voters.

2.5 Positive campaigning

For most voting systems and especially in plurality voting negative campaigning is often a desired tactics to increase chances of a candidate. Multiple votes methods in general motivate for more inclusive campaigns. [AG11] This effect is especially profound for D21 as a potentially winning candidate most likely needs to appeal not only to her primary voters, but also to voters for whom she is the second (or even third) choice, and thus needs to focus on a much broader spectrum of the electorate. For D21, candidates who hold similar political views do not need (and it is no tactical) to directly compete with each other. On the contrary, we will often encounter cases where candidates emphasize qualities of their opponents, or at least part of their political program, in order to get votes from some of their primary voters. Purely negative campaigning could also provoke resentment among voters who could punish a negative campaigner with a minus vote.

For the reasons mentioned above, D21 will lead to a cleaner political environment where candidates could no longer denigrate other candidates not to lose the additional votes of their primary voters. We expect constructive discussions during positive political campaigns, which could also bring more voters to the polls, a clearly very desirable feature for any democratic society.

2.6 Social choice criteria

The well-known Arrow's Impossibility Theorem [Arr51] states that, with at least three distinct alternatives, no voting system can convert the preferences of individuals into a community-wide ranking which fulfills the following natural criteria:

- *unrestricted domain* – all preferences of all voters are allowed,
- *non-dictatorship* – no single person decides by himself or herself,
- *Pareto efficiency* – it is not possible to make one individual better off without making anybody else worse off, i.e., if everybody prefers A over B, then the voting systems must also rank A over B,

- *independence of irrelevant alternatives* (IIA) – the ranking of A over B should depend only on individual preferences between A and B.[6]

If we examine different social choice criteria individually and discuss the conditions under which each may be violated, we can see that D21 performs exceedingly well. We will analyze the very restrictive *Condorcet winner* (CW) criterion that is usually considered desirable. CW requires that if there exists an alternative that would win a head-to-head contest between itself and any other choice, the voting method must always choose that alternative.

The CW criterion is not satisfied by most voting systems. For the most common voting systems such as FPTP or two-round majoritarian system, a violation appears similar to the example provided in Section 2.2, revealing major weaknesses of FPTP and similar majoritarian systems. (A commonly cited example is the French presidential election of 2003, when the likely Condorcet winner, Socialist Lionel Jospin, was eliminated in the first round of voting.)

D21 does not satisfy CW for a fundamentally different reason than FPTP. Consider three alternatives A, B, C, where $A \succ B \succ C$ for $n+1$ voters, and $B \succ C \succ A$ for n voters, $n \in \mathcal{N}$, $n \geq 2$. The Condorcet winner is A, while the D21 winner is B as soon as at least two of the latter voters give more than one vote. B is also the winner in the *Borda count*[7] voting system.

Here we see that the Condorcet winner does not maximize overall social utility. When candidate A wins, all $n+1$ voters are gratified to have their first choice selected, whereas for all n voters, their least-favorite candidate has won! Under D21 (and Borda count), which selects candidate B, n voters now have their first-choice candidate, $n+1$ voters have their second choice, and no voter has their least-favorite choice. Thus, from a social utility (or merely common-sense) point of view, the choice of B is more rational, especially for larger n. This simple example illustrates that the CW criterion does not always lead to the most reasonable result (in the extreme can lead to the "tyranny of majority"), and D21 reflects this insight.

Note that this example also violates the IIA from Arrow's theorem since candidate C is the *non-irrelevant alternative* that changes the voting result for D21 (and Borda count). And yet, the change is desirable from the perspective of overall social utility.

A similar analysis can be done for wide range of other social choice criteria.

3 D21 vs. other voting systems

Interest in alternative to one-vote methods started as early as the eighteenth century. The earliest was Borda's 1781 "Method of Marks," which was put forth

[6] We will see further that, unlike the previous three, IIA might not be a desired criteria from the same reason as might not be the Condorcet winner criterion.

[7] In the Borda count system, devised in 1770 by a French mathematician of that name, voters rank candidates in order of preference. Each candidate receives a number of points corresponding to the number of candidates ranked lower that he or she. A candidate with the most points is the winner. Though Borda count proves to be a more consensus-based voting system than FPTP, it does not benefit from the *effect of more votes*.

as a means of obtaining a consensus of opinions by voter ranking. [CK90] Borda recommended that in an election among three candidates a winner should be selected based on the greatest number of total points assigned by voters. [BF78] A few years later, Borda's contemporary, Condorcet, in 1785 proposed a new way of vote tabulation. He argued that the winner of an election should be that candidate preferred by a simple majority of votes to each other one by winning a majority of votes in pairwise elections between all of those running. [You88] As subsequent work has shown, neither method is ideal. The Borda count has heavy informational requirements and the selection of a winner may be sensitive to which candidates are included in the election. On the other hand, the Condorcet method may not produce any winner at all since majority voting can produce cyclical outcomes. However, both Borda and Condorcet gave a start to the ranked voting method. Since their time, figures such as Kemeny[8], Dodgson[9], and Copeland[10] have also proposed alternative procedures based on voter rankings.

3.1 Instant-runoff voting

One procedure that has already been used in political elections is the *Single transferable vote (STV)* system, where voters rank candidates in order of preference on a scale equal to the number of the running candidates, ranking as many as they choose. [Tid95] When STV is used for single-winner elections it is known as instant-runoff voting (IRV), or occasionally "alternative vote", "transferable vote", or "preferential voting".

Though some jurisdictions have recently adopted STV, scholars have identified systematic problems with it. They noted that these flaws are hidden and can go unnoticed by the electorate. Endersby and Towle express doubts "that STV systems, as currently established, would satisfy reformers who were fully informed of their potential flaws." [ET14] STV is a complex and time-consuming method. Examples show that it is hard to explain to the voters the counting process, which makes it appear less transparent to voters.

Proponents point to significant advantages to procedures that employ rankings. One is that voters may deviate from an expression of single party and/or candidate preference. [Mar07] However, research shows that introduction of STV did not produce the desired outcomes of encouraging voters to vote across party lines; promote diversity and proportionality; or increase voter turnout. [Dun] Moreover, STV did not promote cross-bloc electoral collaboration. [CF17]

It is well-known that STV does not satisfy the "monotonicity criterion", i.e. a winning candidate may lose after receiving a strictly better rank from a voter. [FN18] It is also known that the results of STV may significantly depend on

[8]Kemeny's voting scheme is a preferential voting system where a winner is a preferred candidate in a "Kemeny consensus," the preference ranking of the voters. [HSV05] [YL78]

[9]Dodgson's method selects a winner based on the candidate "closest" to the Condorcet winner position. [Rat01] [BTT89]

[10]Copeland's method ranks candidates according to their win-loss scores in pairwise competitions among all candidates. [NR76] [Nur83] [MS97]

how some voters rank candidates at the end of their list, which can often be easily influenced and opens doors to voters manipulation.

We argue that the STV's greatest weakness, especially profound when only one candidate has to be elected, is hidden in the logic of vote calculation, which creates a systematic bias resulting in disproportionate impact of a certain type of voting preference. The primary supporters of the weakest candidates have greater power to influence election results than those whose first choice are more consensual candidates. A second choice of a more consensual voter whose preferred candidate does not drop out in the calculation is not counted, while successive preferences of those voting primarily for the weakest candidates are counted. We demonstrate this, arguably a major and systematic pitfall, by the following example.

Illustration of a pitfall of STV

Suppose that we have four candidates that we denote as V (visionary), C (consensual), P (populist), and E (extremist). By "visionary", we mean to suggest a candidate who may have exceptional personal qualities, but for reasons of background or style appeals to a smaller, more highly educated set of voters; by "consensual", we mean a candidate who may support the same policies as the "visionary" candidate but who appeals to a broader electorate (again, for reasons of style or background). "Populist" and "extremist" follow the definitions given in Section 2.1.1 above.

We assume four types of voters n_V, n_C, n_P, n_E with the following ranks of preferences that we can argue to be plausible:

Type of a voter	Preferences
n_V	$V \succ C \succ P \succ E$
n_C	$C \succ P \succ V \succ E$
n_P	$P \succ C \succ V \succ E$
n_E	$E \succ P \succ C \succ V$

We assume a rather desirable set of preferences for a democratic society

$$\begin{aligned} n_V > & n_C > n_P > n_E, \\ & n_C < n_P + n_E, \\ n_V < & n_C + n_P + n_E. \end{aligned}$$

The three inequalities in the first line represent arguably the best possible order of social preferences that we could desire for a prosperous democratic society. The second line states that the total number of populist and extremist voters is not negligible (in sum higher than the number of consensual voters), which is very realistic. The third line states that the number of visionary voters is less than half of the total, which is also very plausible.

Under D21, with two votes per voter, candidate C is a clear winner, getting either the first or second vote of every voter except those with the most extreme preferences. What about IRV?

Since no candidate receives more than half of all votes, the weakest candidate E drops and the second preferences of extreme voters n_E are counted as votes for P. This illustrates a key flaw of IRV noted above. In the second calculation, the consensual candidate C is eliminated since $n_P+n_E > n_C$. The second choice of extremist voters together with the first choice of populist voters decide, while the preferences of visionary voters have no effect since C is the second choice for them and the V candidate is still in the running. With C eliminated and $n_V < n_C+n_P+n_E$, the contest has become a "battle" where demagogy defeats intellect, a result suggestive of an anti-visionary or a pro-populism mood in society.

Note that visionary voters had no effect on the result at any point in the calculation even though they are the largest group. In fact, the result would be different with C winning if we decrease the number of visionary voters so that $n_C > n_V > n_P + n_E - n_C$, which again shows the possible logical inconsistency of IRV.

For a specific illustration, imagine a minimalist scenario in which $n_V = 8$, $n_C = 4$, $n_P = 3$, $n_E = 2$. The results for D21 voting would be (with a slight abuse of notation) $C = 15$, $P = 9$, $V = 8$, $E = 2$ with candidate C as the clear winner. The IRV algorithm first eliminates E, then since $4 = n_C < n_P+n_E = 5$, the clear D21-winner C is eliminated, with the result that P wins as $8 = n_V < n_C + n_P + n_E = 9$. Of course, one can choose different numerical illustrations such as $n_V = 73$, $n_C = 37$, $n_P = 33$, $n_E = 7$.

While there are many serious anomalies of IRV well illustrated in literature [FB83], our example further exposes a wide range of faults of STV of major practical significance. The resulting bias favoring certain voting preferences is inherently present and may often negatively influence the quality of the selection of candidates in common practical applications of STV.

3.2 Approval voting

It is worth mentioning that AV proposed in 1970s by Weber was not then a new or a unique method, as it was used for five hundred years, from the 13th to 18th centuries, in Venice. [Lin86] This method of voting was also introduced in local elections in Russia during the reign of Catherine the Great, in the 18th century. An urn, covered with a cloth, was set out for each candidate, with two compartments: "Elect" and "Not Elect." Voters voted by placing a ball in one of the compartments of the urn. [Ale05]

The ballot under an AV method is similar to that under plurality due to the absence of required ranking of the available option. A voter puts either "Yes" or "No" next to each candidate running in the election applying the minimal scale $\{0, 1\}$. Voters can approve as many candidates, with no more than one vote for each. Under AV, a winner is a candidate with the greatest approval total. [BF78]

AV was explored by Brams and Fishburn in the academic literature in the late seventies and has been since adopted by a number of scientific and professional societies. [BF05] Though it has advantages over a one-vote method, it

also has weaknesses. Brams and Fishburn argue that AV's advantages lie in its tendency to find consensus candidates while being simple in practice. [BF78] [Moh11] Their most recent works established AV as "flexible" for expressing opinions; immune against negative campaigning; favorable to a Condorcet winner; and successful in encouraging voter turnout and minority representation. Myerson and Weber also point to its resistance to strategic manipulation. [BF05] [MW93] [Núñ14] Unlike methods requiring rankings as inputs, it is simple for voters to understand. However, Balinki and Laraki indicate that the AV's two-level scale of measurement is so "unnecessarily restrictive as to be unnatural," for expressing complex evaluations by voters, which may be restrictive in some expert-voting applications. [BL11] In addition, intensity of voter preference will vary depending on how many candidates the voter chooses to approve. Moreover, when compared with the plurality rule in the general setting, approval voting is shown to be susceptible to the same kinds of problems as the plurality rule, including the possibility of "non-majoritarian outcomes, failure to elect the Condorcet winner and existence of spoiler candidates." [DO06]

Like D21, AV benefits from the consensus-producing *effect of more votes* per voter. Here the conceptual difference between "highest consensus" and "highest overall utility" is that whereas "consensus" simply measures the number of voters who are satisfied or dissatisfied with a given outcome (a binary "yes/no" measurement), "utility" factors in the strength of satisfaction or dissatisfaction with that outcome, based on the preferences of each voter. The "highest consensus" candidate may often, but not always, be the candidate producing the highest overall social utility as well.

The central weaknesses of AV is its tendency merely to favor candidates who offend the fewest number of voters. These candidates may thus maximize consensus (in terms of the number of voters who favor rather than disfavor them), while not at all maximizing overall utility (because their support from these voters is relatively tepid). We conclude that AV may tend to result in the selection of a "merely inoffensive" candidate to win, one whom many voters are "fine with" despite not taking any strong positions or showing noteworthy qualities of leadership. By optimizing the number of plus-votes in D21 we can find the right balance between consensus and choosing strong preferences, so D21 motivates towards critical thinking.

Given a non-trivial number of candidates, the "scarcity" of votes which are systematically capped in D21 further decreases the motivation for *tactical voting*. Under D21, the *Burr dilemma*[11] is also less likely. For example, the relative "scarcity" of votes under D21 makes people less likely to consider another "merely inoffensive" candidate to be a competition for their favorite candidate, and thus are less likely to resort to tactical voting.

[11] When two candidates, A and B, appear to be the "front-runners", voters are strategically motivated to approve one but not the other. This generates a "vote split" between A and B, which could permit a far-less-favored candidate C to win – as was the case in the U.S. election of 1800, when the less-favored Aaron Burr benefited temporarily from a deadlock between the more-favored John Adams and Thomas Jefferson. Of course, this theoretical weakness is still small compared to the undesirable properties of first-pass-the-post or similar systems.

We further expect that D21 is likely to increase voter participation significantly more than AV. In the case of AV a voter has no incentive to prioritize any candidate over any other – there is no limit to the number of candidates he or she can select, and no way to distinguish strong from weak support. A limited number of votes thus strengthens the "gaming" aspect of voting, motivating voters to carefully consider and discuss their limited set of choices, including the minus vote, to determine how best to distribute their votes.

Evaluative Voting

Another cardinal voting system, an extension of AV, is Evaluative Voting (EV), also referred to as Score Voting, where voters grade candidates on a numerical scale with the winning candidate determined by the sum of the grades they receive. [RT04] Though its proponents define this method as "extremely expressive" [12], the results of several experimental studies demonstrate that its arbitrary scale which can range from $\{0, 1, 2\}$ or $\{0, 1, \cdots, 12\}$, ultimately impacts the outcome. [Bau+14] [Ige+16]

While EV has been put forth with the goal of improving democratic procedures, and hence strengthening such institutions, these systems may introduce many other shortcomings into the democratic processes. We note that D21 provides features meant to address these gaps and weaknesses.

4 Conclusion

On a practical basis, it would be difficult to conceive of a scenario under which D21 (with or without the minus-vote feature) would yield a result inconsistent with the maximum overall social utility. D21 offers a new level of theoretical consistency and correctness, and at the same time has already proven itself to be as promising in practice as it is conceptually. D21 is a voting system for the 21st century, as revolutionary, we argue, as an upgrade from DOS to Windows. Ultimately, Winston Churchill was mistaken – because he only knew of Democracy version 1.0, with a single vote preference. All great ideas must evolve to survive, and the author believes that D21 represents a next step toward the fulfillment of democracy's promise for humankind. The consensus and positive campaigning promoted by voting system D21 might be the cure for today's global problems of separated society.

[12] "Score Voting." The Center for Election Science, accessed at https://electology.org/score-voting, accessed November 15 2018.

Attachment 1: Effect of minus vote (proof)

Let's denote the utility of the left-wing voter gained by electing the candidate L^+ as $U(L^+)$. The utility of the left-wing voter gained from all individual candidates is then the following:

$$U(L^+) > U(L^-) > U(R^+) > U(R^-).$$

For the right-wing voter, it is similar:

$$U(R^+) > U(R^-) > U(L^+) > U(L^-).$$

Let's also mark the number of right-wing voters as n_p and the number of left-wing voters as n_l and the number of all voters as $n = n_l + n_p$.

Without the loss of generality, let us presume that there are more voters of the left-wing than voters of the right-wing, i.e. $n_l > n_p$. (In the inverse case, all consequent considerations are valid with the conversion of indexes l and p.)

We will analyze two possible scenarios:

1. The voter gives two plus-votes.
2. The voter gives two plus-votes and one minus-vote.

In both cases the two candidates with the highest summation will advance (plus-vote counting as 1 point, minus-vote as -1 point).

Scenario without minus-vote

In the case that the voter does not distribute his minus-votes, the result is evident because all voters vote unanimously. We can see results in the Table 1. Because $n_l > n_p$ both left-wing candidates will advance.

Candidate	Votes
R^+	n_p
R^-	n_p
L^+	n_l
L^-	n_l

Table 1: Number of votes in the model without minus-vote.

In the inverse scenario, where right-wing voters outnumber their left-wing counterparts, both of the right-wing candidates will advance.

Let us remind that n is the number of all voters. Let's mark the share of right-wing voters as q and we can express it as

$$q = \frac{n_p}{n}.$$

The share of left-wing voters is complementary up to 1 (that is, $1 - q$) and for this case it holds

$$1 - q = \frac{n_l}{n}.$$

Then we can express the result in this model depending on the share of voters of the right-wing q in the Table 2.

Right-wing share	Advanced
$q < \frac{1}{2}$	L^+ a L^-
$q > \frac{1}{2}$	R^+ a R^-

Table 2: Results of the model without minus-vote.

Model with minus-vote

In the case that the voter distributes one minus-vote, the analysis of the situation is more complicated. Let us remind that the voter prefers criterion of unanimous voting to the criterion of corruption. In this case, the voter divides his two plus-votes between two candidates of his party. It is obvious that he will give his minus-vote to the candidate of the second party – however, it is not necessarily clear to which candidate. In the case that the voter doesn't vote tactically, he will give his vote to the corrupted candidate. We can see the resulting number of votes in the Table 3.

Candidate	Votes
R^+	n_p
R^-	$n_p - n_l$
L^+	n_l
L^-	$n_l - n_p$

Table 3: Number of votes in the model with minus-vote without tactical voting.

The first one elected is the candidate L^+. In the case of the second candidate it depends whether $n_l - n_p < n_p$. This inequality is an equivalent to $n_p/n_l > 1/2$. If we relate inequality to the share of the right-wing q, we will get

$$q > \frac{1}{3}.$$

If such inequality is fulfilled, the candidate R^+ advances as the second one. If it is the other way round (left-wing strongly prevails), the second elected representative will become the candidate L^-.

The results of the vote is described in the Table 4.

Right-wing share	Advanced
$q < \frac{1}{3}$	L^+ and L^-
$\frac{1}{3} < q < \frac{2}{3}$	L^+ and R^+
$q > \frac{2}{3}$	R^+ and R^-

Table 4: Results of the model with minus-vote without tactical voting.

In the worst case, each voter will use tactical voting (which is, however, not very realistic). In this case, the best option for supporters of the left-wing party is to choose the so-called "dominant strategy",[13] i.e. to distribute minus-votes equally between both two candidates of the right-wing. To do so, the left-wing voters weaken both opposing-party candidates equally, to the benefit of their corrupted candidate. The dominant strategy for the right-wing is to give all minus-votes to corrupted candidate of the left-wing and therefore maximize the chance of their own honest candidate. We can see resulted number of votes in Table 5.

Candidate	Votes
R^+	$n_p - n_l/2$
R^-	$n_p - n_l/2$
L^+	n_l
L^-	$n_l - n_p$

Table 5: Number of votes in the model with minus-vote in the worst case.

The first elected candidate is obviously L^+. Right-wing candidates have approximately the same result of votes. One can assume that the candidate R^+ will get somewhat fewer minus-votes and will have better result than candidate R^-. The right-wing candidate R^+ is elected when $n_p - n_l/2 > n_l - n_p$, which is an equivalent to $n_p/n_l > 3/4$ and expressed in relation to the share of right-wing voters

$$q > \frac{3}{7}.$$

General results are described in the Table 6.

Right-wing share	Advanced
$q < \frac{3}{7}$	L^+ a L^-
$\frac{3}{7} < q < \frac{4}{7}$	L^+ a R^+
$q > \frac{4}{7}$	R^+ a R^-

Table 6: Results of the model with minus-vote in the worst case.

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